Version 1.0

General Certificate of Education (A-level)
June 2012

Mathematics MFP3

(Specification 6360)

Further Pure 3

Mark Scheme

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Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
√ or ft or F	follow through from previous incorrect result
CAO	correct answer only CSO
	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
–*x* EE	deduct *x* marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

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Q	Solution	Marks	Total	Comments
1	$k_1 = 0.25\times\left(\sqrt{2\times 2}+\sqrt{9}\right)$ (=1.25)	M1		PI. May see within given formula
	k_2 = 0.25 f (2.25, 9 +1.25)			Either k_2 =0.25 f (2.25, 10.25) stated/used or
	$k_2 = 0.25\times\left(\sqrt{2\times 2.25}+\sqrt{9+1.25}\right)$	M1		$k_2 = 0.25\times\left(\sqrt{2\times 2.25}+\sqrt{9+\text{c's } k_1}\right)$ PI. May see within given formula
	k_2 = 1.33(072…)	A1		k_2 = 1.33(072…) 2 dp or better PI by later work
	y (2.25) = y (2) + $\frac{1}{2}[k_1+k_2]$			
	= 9 + 0.5 [1.25 + 1.33 (072…)] = 9 + 0.5×2.58 (072…)	m1		Dep on previous two Ms and y (2) = 9 and numerical values for k's
	y (2.25) = 10.29036… = 10.29 (to 2 dp)	A1	5	CAO Must be 10.29
	Total		**5**	
2(a)	$\sin 2x = 2x-\frac{(2x)^3}{3!}+\frac{(2x)^5}{5!}...$ $= 2x-\frac{4}{3}x^3+\frac{4}{15}x^5$	B1	1	Accept ACF even if unsimplified
(b)	$\lim_{x\to 0}\left[\frac{2x-\sin 2x}{x^2\ln(1+kx)}\right]$			
	$=\lim_{x\to 0}\frac{2x-(2x-\frac{4}{3}x^3+\frac{4}{15}x^5...)}{x^2\left(kx-\frac{(kx)^2}{2}+...\right)}$	M1		Using series expansions.
		B1		Expansion of ln (1 + kx) = kx (−…)
	$=\lim_{x\to 0}\left[\frac{\frac{4}{3}x^3-\frac{4}{15}x^5+..}{kx^3-\frac{k^2}{2}x^4}\right]$			
	$=\lim_{x\to 0}\left[\frac{\frac{4}{3}-O(x^2)}{k-O(x)}\right]$	m1		Dividing numerator and0 denominator by x^3 to get constant term in each. Must be at least a total of 3 terms divided by x^3
	$\frac{4}{3k}=16\Rightarrow k=\frac{1}{12}$	A1	4	OE exact value. Dep on numerator being of form 4/3(OE) + λx^2... ($\lambda\neq 0$) and denominator being of form $k+\mu x$.. ($\mu\neq 0$) before limit taken
	Total		**5**	

Q	Solution	Marks	Total	Comments
3	Area = $\frac{1}{2}\int \left(2\sqrt{1+\tan\theta}\right)^2 \ (\mathrm{d}\theta)$	M1		Use of $\frac{1}{2}\int r^2 \ (\mathrm{d}\theta)$
	$= \frac{1}{2}\int_{-\frac{\pi}{4}}^{0} 4(1+\tan\theta)\ \mathrm{d}\theta$	B1		Correct limits. If any contradiction use the limits at the substitution stage
	$= 2[\theta + \ln\sec\theta]_{-\frac{\pi}{4}}^{0}$	B1		$\int k(1+\tan\theta)(\mathrm{d}\theta) = k(\theta + \ln\sec\theta)$ ACF ft on c's k
	$= 2\left\{0 - \left[-\frac{\pi}{4} + \ln\sec\left(-\frac{\pi}{4}\right)\right]\right\}$			
	$= 2\left(\frac{\pi}{4} - \ln\sqrt{2}\right) = \frac{\pi}{2} - 2\ln\sqrt{2} = \frac{\pi}{2} - \ln 2$	A1	4	CSO AG
	Total		**4**	
4(a)	IF is $\mathrm{e}^{\int \frac{4}{2x+1}\mathrm{d}x}$	M1		PI
	$\mathrm{e}^{2\ln(2x+1)\ (+c)} = \mathrm{e}^{\ln(2x+1)^2\ (+c)}$	A1		Either O.E. Condone missing '+ c'
	$= (A)(2x+1)^2$	A1F		Ft on earlier $\mathrm{e}^{\lambda \ln(2x+1)}$, condone missing '$A$'
	$(2x+1)^2\frac{\mathrm{d}y}{\mathrm{d}x} + 4(2x+1)y = 4(2x+1)^7$			
	$\frac{\mathrm{d}}{\mathrm{d}x}[(2x+1)^2 y] = 4(2x+1)^7$	M1		LHS as d/dx (y × c's IF) PI **and** also RHS of form $p\,(2x+1)^q$
	$(2x+1)^2 y = \int 4(2x+1)^7\ \mathrm{d}x$	A1		
	$(2x+1)^2 y = \frac{1}{4}(2x+1)^8\ (+c)$	B1F		Correct integration of $p\,(2x+1)^q$ to $\frac{p(2x+1)^{q+1}}{2(q+1)}$ (+ c) ft for q>2 only
	(GS): $y = \frac{1}{4}(2x+1)^6 + c(2x+1)^{-2}$	A1	7	Must be in the form $y = \mathrm{f}(x)$, where $\mathrm{f}(x)$ is ACF
(b)	$y = \frac{1}{4}(2x+1)^6 + c(2x+1)^{-2}$			
	When $x = 0$, $\frac{\mathrm{d}y}{\mathrm{d}x} = 0$	M1		Using boundary condition $x = 0$, $\frac{\mathrm{d}y}{\mathrm{d}x} = 0$ and c's GS in (a) towards obtaining a value for c
	$\Rightarrow y = 1\ \left[\frac{\mathrm{d}y}{\mathrm{d}x} = 3(2x+1)^5 - 4c(2x+1)^{-3}\right]$	B1		Either $y = 1$ or correct expression for dy/dx in terms of x only
	$\Rightarrow c = \frac{3}{4}$ so $y = \frac{1}{4}(2x+1)^6 + \frac{3}{4}(2x+1)^{-2}$	A1	3	CSO
	Total		**10**	

Q	Solution	Marks	Total	Comments
5(a)	$\int x^2 e^{-x} dx = -x^2e^{-x} - \int -2xe^{-x}dx$	M1 A1		$kx^2e^{-x} - \int 2kx\,e^{-x}\,(dx)$ for $k = \pm 1$
	$= -x^2e^{-x} + 2\{-xe^{-x} - \int -e^{-x}dx\}$	m1		$\int x\,e^{-x}dx = \lambda xe^{-x} - \int \lambda e^{-x}(dx)$ for $\lambda = \pm 1$ in 2nd application of integration by parts
	$= -x^2e^{-x} - 2xe^{-x} - 2e^{-x}\,(+c)$	A1	4	Condone absence of + c
(b)	$I = \int_0^\infty x^2e^{-x}\,dx = \lim_{a\to\infty}\int_0^a x^2e^{-x}\,dx$ $\lim_{a\to\infty}\{-a^2e^{-a} - 2ae^{-a} - 2e^{-a}\} - [-2]$	M1		F(a) – F(0) with an indication of limit '$a \to \infty$' and F(x) containing at least one $x^n e^{-x}$, $n > 0$ term
	$\lim_{a\to\infty} a^k e^{-a} = 0$, (k>0)	E1		For general statement or specific statement for either k = 1 or k = 2
	$\int_0^\infty x^2e^{-x}\,dx = 2$	A1	2	CSO
	Total			
6(a)	$y = \ln(1+\sin x)$, $\frac{dy}{dx} = \frac{1}{1+\sin x} \times (\cos x)$	M1 A1	2	Chain rule OE ACF eg $e^{-y}\cos x$
(b)	$\left(\frac{d^2y}{dx^2} = \right)\frac{(1+\sin x)(-\sin x) - \cos x(\cos x)}{(1+\sin x)^2}$	M1 A1		Quotient rule OE, with u and v non constant ACF
	$\frac{d^2y}{dx^2} = \frac{-\sin x - 1}{(1+\sin x)^2} = \frac{-1}{1+\sin x} = \frac{-1}{e^y} = -e^{-y}$	A1	3	CSO AG Completion must be convincing
(c)	$\frac{d^3y}{dx^3} = e^{-y}\frac{dy}{dx}$	B1		ACF for $\frac{d^3y}{dx^3}$
	$\frac{d^4y}{dx^4} = -e^{-y}\left(\frac{dy}{dx}\right)^2 + e^{-y}\frac{d^2y}{dx^2}$	M1		Product rule OE and chain rule
	$\frac{d^4y}{dx^4} = -e^{-y}\left(\frac{dy}{dx}\right)^2 - (e^{-y})^2$	A1	3	OE in terms of e^{-y} and $\frac{dy}{dx}$ only
(d)	$y(0) = 0$; $y'(0) = 1$; $y''(0) = -1$;	B1F		Ft only for $y'(0)$; other two values must be correct
	$y(x) \approx$ $y(0) + xy'(0) + \frac{x^2}{2}y''(0) + \frac{x^3}{3!}y'''(0) + \frac{x^4}{4!}y^{(iv)}(0)$ $y'''(0) = 1$; $y^{(iv)}(0) = -2$	M1		Maclaurin's theorem applied with numerical values for $y'(0)$, $y''(0)$, $y'''(0)$ and $y^{(iv)}(0)$. M0 if missing an expression for any one of the 1st, 3rd or 4th derivatives
	$\ln(1+\sin x) \approx x - \frac{1}{2}x^2 + \frac{1}{6}x^3 - \frac{1}{12}x^4 \ldots$	A1	3	A0 if FIW
	Total		**11**	

Q	Solution	Marks	Total	Comments
7(a)	$\frac{\mathrm{d}x}{\mathrm{d}t}\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}y}{\mathrm{d}t}$	M1		OE Relevant chain rule eg $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}t}{\mathrm{d}x}\frac{\mathrm{d}y}{\mathrm{d}t}$
	$\mathrm{e}^{t}\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}y}{\mathrm{d}t} \Rightarrow x\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}y}{\mathrm{d}t}$	A1		OE eg $\frac{\mathrm{d}y}{\mathrm{d}x} = \mathrm{e}^{-t}\frac{\mathrm{d}y}{\mathrm{d}t}$
	$\frac{\mathrm{d}}{\mathrm{d}t}\left(x\frac{\mathrm{d}y}{\mathrm{d}x}\right) = \frac{\mathrm{d}^2y}{\mathrm{d}t^2}$; $\frac{\mathrm{d}x}{\mathrm{d}t}\frac{\mathrm{d}}{\mathrm{d}x}\left(x\frac{\mathrm{d}y}{\mathrm{d}x}\right) = \frac{\mathrm{d}^2y}{\mathrm{d}t^2}$	M1		OE. Valid 1st stage to differentiate $x\,y'(x)$ oe with respect to t or to differentiate $x^{-1}y'(t)$ oe with respect to x.
	$\frac{\mathrm{d}x}{\mathrm{d}t}\left(\frac{\mathrm{d}y}{\mathrm{d}x} + x\frac{\mathrm{d}^2y}{\mathrm{d}x^2}\right) = \frac{\mathrm{d}^2y}{\mathrm{d}t^2}$	m1		Product rule (dep on previous M)
	$x^2\frac{\mathrm{d}^2y}{\mathrm{d}x^2} + x\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}^2y}{\mathrm{d}t^2}$	A1		OE eg $\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = \mathrm{e}^{-t}\left[-\mathrm{e}^{-t}\frac{\mathrm{d}y}{\mathrm{d}t} + \mathrm{e}^{-t}\frac{\mathrm{d}^2y}{\mathrm{d}x^2}\right]$ {Note: e^{-t} could be replaced by $1/x$}
	$x^2\frac{\mathrm{d}^2y}{\mathrm{d}x^2} - 4x\frac{\mathrm{d}y}{\mathrm{d}x} + 6y = 3 + 20\sin(\ln x)$ becomes $\frac{\mathrm{d}^2y}{\mathrm{d}t^2} - x\frac{\mathrm{d}y}{\mathrm{d}x} - 4x\frac{\mathrm{d}y}{\mathrm{d}x} + 6y = 3 + 20\sin(\ln x)$			
	$\Rightarrow \frac{\mathrm{d}^2y}{\mathrm{d}t^2} - 5\frac{\mathrm{d}y}{\mathrm{d}t} + 6y = 3 + 20\sin(\ln \mathrm{e}^{t})$	m1		Substitution to reach a 'one-step away' stage for LHS. Dep on previous M M m
	$\Rightarrow \frac{\mathrm{d}^2y}{\mathrm{d}t^2} - 5\frac{\mathrm{d}y}{\mathrm{d}t} + 6y = 3 + 20\sin t$	A1	7	CSO AG
(b)	Auxl eqn $m^2 - 5m + 6 = 0$	M1		PI
	$(m-2)(m-3) = 0$, $m = 2, 3$	A1		
	CF: $(y_C =)\ A\mathrm{e}^{2t} + B\mathrm{e}^{3t}$	A1F		Ft wrong values of m provided 2 real roots, and 2 arb. constants in CF. Condone x for t here
	P.Int. Try $(y_P =)\ a + b\sin t + c\cos t$	M1		Condone 'a' missing here
	$(y'(t)=)\ b\cos t - c\sin t$	A1		
	$(y''(t)=)\ -b\sin t - c\cos t$	A1F		ft can be consistent sign error(s)
	Substitute into DE gives	M1		Substitution and comparing coefficients at least once
	$a = 0.5$	B1		
	$5c + 5b = 20$ and $5c - 5b = 0$	A1		OE
	$b = c = 2$	A1		
	GS $(y =)\ A\mathrm{e}^{2t} + B\mathrm{e}^{3t} + 2\sin t + 2\cos t + \frac{1}{2}$	B1F	11	Ft on c's CF + PI, provided PI is non-zero and CF has two arbitrary constants and RHS is fn of t only
(c)	$y = Ax^2 + Bx^3 + 2\sin(\ln x) + 2\cos(\ln x) + 0.5$	B1	1	CAO
	Total		**19**	

Q	Solution	Marks	Total	Comments
8(a)	$xy = 8 \Rightarrow\ r\cos\theta\ r\sin\theta\ = 8$	M1		
	$\frac{1}{2}r^2\sin 2\theta = 8$	m1		Use of $\sin 2\theta = 2\sin\theta\cos\theta$
	$r^2 = \frac{16}{\sin 2\theta} = 16\,\text{cosec}\,2\theta$	A1	3	AG Completion
(b)(i)	(At N, r is a minimum $\Rightarrow \sin 2\theta = 1$) $N\left(4, \frac{\pi}{4}\right)$	B1B1	2	B1 for each correct coordinate.
(ii)	At pts of intersection, $\left(4\sqrt{2}\right)^2 = 16\,\text{cosec}\,2\theta$	M1		
	$\sin 2\theta = \frac{1}{2}$	A1		PI by $\text{cosec}\,2\theta = 2$ and a correct exact or 3SF value for 2θ or θ
	$2\theta = \frac{\pi}{6}$, $\frac{5\pi}{6}$	A1		PI OE exact values
	$\left(4\sqrt{2}\ ,\ \frac{\pi}{12}\right)$ $\left(4\sqrt{2}\ ,\ \frac{5\pi}{12}\right)$	A1	4	Both required, written in correct order
(iii)	$\angle POQ = \frac{5\pi}{12} - \frac{\pi}{12} = \frac{\pi}{3}$ **or** $\angle\ PON = \frac{\pi}{6}\ (= \angle QON)$	B1F		Ft on c's $\theta_P, \theta_Q, \theta_N$ as appropriate OE
	$PN^2 = \left(4\sqrt{2}\right)^2 + \left(r_N\right)^2 - 2\left(4\sqrt{2}\right) r_N \cos\left(\frac{1}{2}POQ\right)$ **or** $PT = 4\sqrt{2}\sin\left(\frac{1}{2}POQ\right)$ **or** $PT = \frac{1}{2}\times 4\sqrt{2}$ **or** $NT = 4\sqrt{2}\cos\left(\frac{1}{2}POQ\right) - r_N$	M1		Finding the lengths of **two** unequal sides of ΔPNQ or ΔPNT or ΔQNT, where T is the point at which ON produced meets PQ. Any valid equivalent methods eg finding $\tan\angle OPN$ or finding $\sin\angle ONP$.
	$PN = \sqrt{\left(48 - 16\sqrt{6}\right)}$ [=2.96(7855…)] = NQ **or** $PT = 2\sqrt{2}$ [=2.82(8427…)] **or** $PQ = 4\sqrt{2}$ **or** $NT = 2\sqrt{6} - 4$ [=0.898(979…)]	A1		Two correct unequal lengths of sides of ΔPNQ or ΔPNT or ΔQNT PI OE eg $\tan\angle OPN = 1/\left(2\sqrt{2} - \sqrt{3}\right)$ or $\sin\angle ONP = 2\sqrt{2}/(\sqrt{48 - 16\sqrt{6}})$
	$\tan\frac{\alpha}{2} = \frac{PT}{NT} = \frac{2\sqrt{2}}{2\sqrt{6} - 4}$ [=3.14626…] OE **or** $\frac{\alpha}{2} = \frac{\pi}{2} - \left[\frac{\pi}{3} - \tan^{-1}\left(\frac{1}{2\sqrt{2} - \sqrt{3}}\right)\right]$ **or** $32 = 2PN^2(1 - \cos\alpha) \Rightarrow 1 - \cos\alpha = \frac{1}{3 - \sqrt{6}}$	m1		Valid method to reach an eqn in α (or in $\frac{\alpha}{2}$) only; dep on prev M but not on prev A. Alternative choosing eg obtuse ONP then $\frac{\alpha}{2} = \pi - 1.87(85...)$
	$\frac{\alpha}{2} = 1.263056...$; $\alpha = 2.5261...2.53$ to 3sf	A1	5	2.53… Condone >3sf.
	Total		**14**	
	TOTAL		**75**	